

Finance II: Annuities Pre-Work (SOLUTIONS)

Overview:

This session continues the coverage of financial math with financial instruments consisting of a finite series of equal periodic payments.

1. What is the present value of an annuity in which \$2,000 is paid each year for 5 years, assuming a discount rate of 5% and the first payment is received one year from now?

Approach 1: Compute PV for each payment and sum result

$$\begin{aligned}
 \text{PV of first payment} &= 2,000/1.05 = 1,904.76 \\
 \text{PV of second payment} &= 2,000/1.05^2 = 1,814.06 \\
 \text{PV of third payment} &= 2,000/1.05^3 = 1,727.68 \\
 \text{PV of fourth payment} &= 2,000/1.05^4 = 1,645.40 \\
 \text{PV of fifth payment} &= 2,000/1.05^5 = 1,567.05
 \end{aligned}$$

$$\text{PV} = \$8,658.95$$

Approach 2: Use direct formula

$$\begin{aligned}
 \text{PV} &= (2,000/0.05) * [1 - (1/1.05)^5] \\
 &= 40,000 * (1 - 0.7835) \\
 &= 40,000,000 * 0.2165 \\
 &= \$8,658.95
 \end{aligned}$$

2. What is the present value of an annuity in which \$2,000 is paid each year for 5 years, assuming a discount rate of 9% and the first payment is received one year from now? How does the value of this annuity compare to your result from the previous exercise. Why?

Approach 1: Compute PV for each payment and sum result

$$\begin{aligned}
 \text{PV of first payment} &= 2,000/1.09 = 1,834.86 \\
 \text{PV of second payment} &= 2,000/1.09^2 = 1,683.36 \\
 \text{PV of third payment} &= 2,000/1.09^3 = 1,544.37 \\
 \text{PV of fourth payment} &= 2,000/1.09^4 = 1,416.85 \\
 \text{PV of fifth payment} &= 2,000/1.09^5 = 1,299.86
 \end{aligned}$$

$$\text{PV} = \$7,779.30$$

Approach 2: Use direct formula

$$\begin{aligned}
 \text{PV} &= (2,000/0.09) * [1 - (1/1.09)^5] \\
 &= 22,222.22 * (1 - 0.6499) \\
 &= 22,222.22 * 0.3501 \\
 &= \$7,779.30
 \end{aligned}$$

Because of the higher discount rate, note that the annuity value in this exercise is lower than the annuity value in the previous exercise.